

# Hiring Expert Consultants in E-Healthcare: A Two Sided Matching Approach

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**Abstract**—Very often in some censorious healthcare scenario, there may be a need to have some expert consultancies (especially by doctors) that are not available in-house to the hospital. With the advancement in technologies (such as video conferencing, smartphone, etc.), it has become reality that, for the critical medical cases in the hospitals, expert consultants (ECs) from around the world could be hired, who will serve the patients by their physical or virtual presence. Earlier, this interesting healthcare scenario of hiring the ECs (mainly doctors) from outside of the hospitals had been studied with the robust concepts of mechanism design with or without money. We have tried to model the *ECs (mainly doctors) hiring problem* as a two sided matching problem. In this paper, for the first time, to the best of our knowledge, we explore the more *realistic* two sided matching in our set-up, where the members of the two participating communities, namely *patients* and *doctors* are revealing the strict preference ordering over all the members of the opposite community for a stipulated amount of time. We assume that patients and doctors are *strategic* in nature. With the theoretical analysis, we demonstrate that the proposed mechanism that results in *stable* allocation of doctors to patients is *strategy-proof* (or *truthful*) and *optimal*. The proposed mechanism is also validated with exhaustive experiments.

**Index Terms**—E-Healthcare, hiring ECs, DSIC, mechanism design, *stable* allocation.

## I. INTRODUCTION

**T**HE *expert advices* or *consultancies* provided by the expert consultants (ECs) mainly doctors can be thought of as one of the most indispensable events that occurs in the hospital(s) or medical unit(s) on a regular basis. More formally, it is said to be the *crux* of the hospitals and the operation theatres (OTs). Over the past few years, there had been a perplexing growth in the demand of ECs (especially *doctors*) during some critical surgical processes (or operations) that are taking place in the OTs of the hospitals. The unprecedented growth in the demand of the ECs, has made ECs *busy* and *scarce* in nature. It is to be noted that, this unique nature (*i.e.* busy and scarce) of ECs in the healthcare lobby provides an edge to the research community in the healthcare domain to think of: *How to manage or schedule these limited (or scarce) ECs in the OTs of the hospitals, during some censorious healthcare situation?* In order to answer the above coined question, previously, there

had been a spate of research work in the direction of handling the issues of scheduling the in-house ECs especially doctors [1], [2] and nurses [3]–[7] in an efficient and effective manner. In [1], [2], [8], [9] different techniques are discussed and presented to schedule the physicians that are in-house to the hospitals in an efficient way for some critical operations that are taking place in the OTs of the hospitals.

On reviewing the above discussed literature works, we have found that, they have addressed the scenario of scheduling the scarce or more importantly busy resources (such as *doctors*, *nurses* etc.) that are available in-house to the hospitals to the OTs of those hospitals. More importantly, in healthcare domain, one scenario that may be thought of as a challenging issue is, say; in certain critical medical cases, there may be a requirement of some external manpower in the form of ECs (mainly doctors) that are not available in-house to the hospitals. Now, the immediate *natural* question that came in the mind is that, *how to have some external expertise mainly in the form of doctors that are not available in-house to the hospitals?* Surprisingly, literature is very limited for this problem in healthcare domain. First time, this interesting situation of taking expert consultancy from outside of the in-house medical unit during some censorious medical scenario (mainly *surgical* process) was taken care by *Starren et. al.* [10]. Moreover, the introduction of such a pragmatic field of study in the healthcare domain by *Starren et. al.* [10] has given rise to several open questions for the researchers, such as: (a) *which ECs are to be considered as the possible expertise provider in the consultancy arena?* (b) *What incentives policies in the form of perks and facilities are to be presented in-front of the ECs, so as to drag as many ECs as possible in the consultancy arena?*

Answering the above mentioned questions, first time in literature, the idea of hiring ECs (mainly doctors) from outside of the hospitals was endeavoured by *Singh et. al.* [11] under *economic* perspective. In [11], the problem of hiring one or more doctors for a patient from outside of the admitted hospital for some critical operation under monetary environment with the *infinite budget* are addressed. By *monetary environment* it is meant that, money is involved in the set-up by some means. It is assumed that, the ECs are *strategic* in nature. By *strategic* they mean that the ECs may misrepresent their private information (bid value) in order to gain. Due to presence of *strategic* ECs in the consultancy arena, they have modelled the problem with the robust theory of *mechanism design*. The incentives are provided to motivate the doctors to participate in the consultancy arena from around the world. With the consideration that, ECs are having some social connections

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in real life, *Singh et. al.* [12] considered a budgeted setting of the problem in [11] with a goal to maximize the total number of ECs hired for the consultancy procedure for a patient with a *fixed* budget. As opposed to the money involved hiring of ECs as mentioned in [11], [12] another market of hiring ECs can be thought of where the ECs are providing their expertise free of cost. Recently, *Singh et. al.* [13] have addressed this idea, where the expert services are distributed free of cost. For hiring ECs in *monetary free* environment (*i.e.* money is not involved in any sense) they have utilized the idea of one sided strict preference (in this case strict preference from patient side) over the available doctors in the consultancy arena. The novelty that is achieved in [13] is that, each patient is allocated with the best possible doctor from their available respective revealed preference list when his/her turn came. In [13] the model is studied under the realistic viewpoint that the patients are *strategic* in nature and can mis-report their preference list in order to be *better off*.

Similar to the background in [13], in this paper, to the best of our knowledge, first time we have tried to model the *ECs hiring problem* as a two sided preference market. The idea behind studying the *ECs hiring problem* as a more appealing two sided preference market is that, in this environment, the members present in two different communities have the privilege to provide the strict preference ordering over all the available members of the opposite community. For example, in our case, we have two communities (or parties) in the consultancy arena: (a) *Patient party* (b) *Doctor party*. So, each of the members of the *patient party* provide strict preference ordering over all the available members in the *doctor party* and *vice versa*. The scenario that some subset of the patient party and the doctor party provides the preference ordering (not necessarily strict) over the subset of the members of the opposite community is left as our immediate future work.

Our system model consists of  $n$  hospitals as depicted in the left side in Figure 1. In each hospital several patients of different categories (such as *eye surgery*, *cardiovascular surgery*, etc.) are admitted who may need consultancies by the doctors that are either not available completely or partially available and need more consultation that are not present in-house to the hospitals. The patients who need consultancy may belong to different income bars. So, in this scenario, each hospital tries to select the patient from the lowest income bar in a particular category (say  $c_i$  category) who will get the free consultation. Now, the situation is at a particular time several doctors ( $> n$ ) are providing their willingness to impart free consultancy to some patients present in the consultancy arena in a particular category as shown in the right side of the Figure.

1. The third party selects  $n$  doctors out of all the doctors in a particular category as a possible expert consultant. For placing  $n$  doctors in the consultancy arena from the available doctors the *third party* can take the help of the qualification of the doctors and number of successful operations or consultancies given so far by that doctor. Now, in our consultancy arena we have  $n$  patients that need an expert consultancy and  $n$  doctors who are willing to provide their expertise free of cost in a particular category. The  $n$  number of available patients in *patient party* give their strict preference ordering over the

$n$  number of available doctors in *doctor party* and also the  $n$  number of available doctors in *doctor party* give their strict preference ordering over the  $n$  number of available patients in the *patient party* simultaneously for the stipulated amount of time. The doctors may give preferences based on the location where he/she (henceforth he) and the patients are located. The problem is to allocate the doctors to the patients, so that each of the patients and doctors gets their best possible choice.

#### A. Our Contributions

There exists little work in the literature (we have discussed above) that deals with the problem of hiring ECs from outside of the in-house medical unit. However, there is no existing work in this direction that additionally captures the idea of two sided strict preference ordering over the opposite parties *i.e.* in our case, the *patient party* are providing *strict preference* ordering over all the members of *doctor party* and the *doctor party* are providing *strict preference* ordering over all the members of *patient party*. The main contributions of our work are as follows.

- First time we have tried to model the *ECs hiring problem* as a two sided matching problem.
- We propose a *truthful* and *optimal* mechanism; namely *truthful optimal mechanism for hiring expert consultants (TOMHECs)*. We believe that this is the first attempt in the direction of designing a *truthful* mechanism for this interesting class of problem of hiring expert consultants in healthcare domain.
- We establish an upper bound of  $O(kn^2)$  (**Proposition 1**) on the number of iterations required to determine a *stable* allocation for any instance of  $n$  patients and  $m$  doctors such that  $m = n$ .
- We have also proved that for any instance of  $n$  patients and  $n$  doctors the allocation done by TOMHECs results in *stable*, *truthful*, and *optimal* for requesting party.
- A substantial amount of simulation is done to validate the performance of *randomized mechanism for hiring expert consultants (RAMHECs)* and TOMHECs via *optimal* allocation measure.

The remainder of the paper is structured as follows. Section II elucidates the preliminary concepts about scheduling in healthcare domain and the two sided matching. Section III describes our proposed model. Some required definitions are discussed in Section IV. The proposed mechanisms is illustrated in section V. A detailed analysis of the experimental results is carried out in section VI. Finally, conclusions are drawn in Section VII.

## II. PRIOR WORK

The prior art on scheduling of several scarce resources in healthcare domain can be classified into two broad categories: one addressing the scheduling and managing of scarce resources such as *nurses*, *physicians*, *hospitals*, *OTs* etc. that are in-house to the medical units; with the other addressing the research challenges encountered when ECs (*doctors*, *nurses*, etc.) are hired from outside of the hospitals. Our paper can be classified more in the *second* category. While there are several

challenges that still exists in this area of healthcare. Our work finds relevance to the scenario of hiring ECs from outside of the medical unit with two sided preference.

**Scheduling in Healthcare:** [3] [4] [5] [6] [7] [1] [2] literatures addressed the various scheduling problems in healthcare domain. However, most of them are concerned only about scheduling the scarce resources such as *nurses, physicians, etc.* inside the medical unit in an efficient and effective way. In past, the work had been also done in the direction of managing the operation theaters (OTs) and the hospitals during the patient congestion scenario. The work in [14] [15] [16] focuses on the question of: *how to effectively and efficiently plan and schedule the OTs?* In [17] [18] [16] the work has been done for allocating OTs on time to increase operating room efficiency.

Little work has been done on the problem of hiring the ECs from outside the hospital during critical operations for consultancy purpose. *Starren et. al.* [10] first time introduced the notion of taking expert consultancy from outside of the in-house hospital. But this paper left out several other open questions in this scenarios such as: How to motivate the doctors to take part in the consultancy arena as they may be very busy? which doctors can be hired? If the incentives are provided, how much can be offered? Considering these introduced questions and several other issues, in [11], the problem of hiring expert consultants from outside of the hospital for performing critical operation under monetary perspective has been considered. In *Singh et. al* [11] a game theoretic approach to tackle the situation of hiring a doctor from outside of the hospital has been developed. Similar to their set-up discussed in [11], *Singh et. al.* [12] tried to provide an *approximate truthful mechanism* motivated by [19] for hiring  $k$  doctors out of the available  $n$  doctors ( $k < n$ ), such that the total payment made to the ECs do not exceed the total budget of a patient. In the series of research in this direction one of the interesting situations is addressed in [13] that tackle the situation of hiring the doctors from outside of the hospital under *non-monetary* environment. *Singh et. al.* [13] proposed the *optimal* mechanism motivated by [20] [21] to allocate the doctors to the patients admitted to different hospitals based on their strict preference profile over the available doctors. They have introduced this set-up under the consideration that patient's are *strategic* in nature and may misrepresent their preference profile in order to gain. Moreover, it is our claim that till date no literature work has addressed the more appealing scenario, where both the participating community *i.e. patient party* and the *doctor party* reveals the strict preference ordering over all the members of opposite community under their *strategic* nature.

**Two sided matching:** In past, the two sided matching market is very well abstracted by gale and shapley in their seminal paper [20]; using the *college admissions* and *stable marriage problem*. This pioneering work by *Gale & Shapley* received a profound attention from the research community of different domains from around the world because of its vast real world applications. It is to be noted that, the work in [20] is motivated by hospital-resident problem commonly known as *National Resident Matching Problem (NRMP)* [22] [23]. There exists several other Residents-Hospitals systems

in various parts of the world such as: CaRMS [24] in Canada, SPA [25] in Scotland, and JRMP [26] in Japan, that utilizes the concept of *stable marriage problem*. For more detail study in the direction of two-sided matching, we refer [27] [28] [29] and a survey [30] to the readers. Our proposed mechanism is motivated by [20] [31].

### III. SYSTEM MODEL

In this section, we formalize the *non-monetary* mechanism design problem of hiring doctors from outside of the hospitals for the patient(s). We consider the scenario, where there are multiple hospitals say  $n$  given as  $\mathcal{H} = \{h_1, h_2, \dots, h_n\}$ . In each hospital  $h_i \in \mathcal{H}$ , there exists several patients with different diseases (in our case patients and doctors are categorized based on the diseases and areas of expertise respectively.) belonging to different income group that requires the expert consultancies from outside of the admitted hospitals. The set of  $k$  different categories is given as:  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$ . The set of all the admitted patients in different categories to different hospitals is given as  $\mathcal{P} = \{\{p_{1(1)}^{h_1}, \dots, p_{1(h_1)}^{h_1}\}, \dots, \{p_{k(1)}^{h_n}, \dots, p_{k(h_n)}^{h_n}\}\}$ ; where  $p_{i(j)}^{h_k}$  is the patient  $j$  belonging to  $c_i$  category admitted to  $h_k$  hospital. The expression  $\hat{h}_k^j$  in term  $p_{i(j)}^{h_k}$  indicates the total number of patients in hospital  $k$  belonging to  $c_j$  category. Based on some income criteria, a patient from each of the hospitals that requires an expertise from outside are selected and placed into the consultancy arena. On the other hand, there are several doctors having different expertise associated with different hospitals say  $\mathcal{H} = \{H_1, H_2, \dots, H_n\}$ . The set of all the available doctors is given as  $\mathcal{D} = \{\{d_{1(1)}^{H_1}, \dots, d_{1(h_1)}^{H_1}\}, \dots, \{d_{k(1)}^{H_n}, \dots, d_{k(h_n)}^{H_n}\}\}$ ; where  $d_{i(j)}^{H_k}$  is the doctor  $j$  belonging to  $c_i$  category associated to  $H_k$  hospital. The expression  $\mathcal{H}_k^j$  in term  $d_{i(j)}^{H_k}$  indicates the total number of doctors associated with hospital  $k$  in  $c_j$  category.

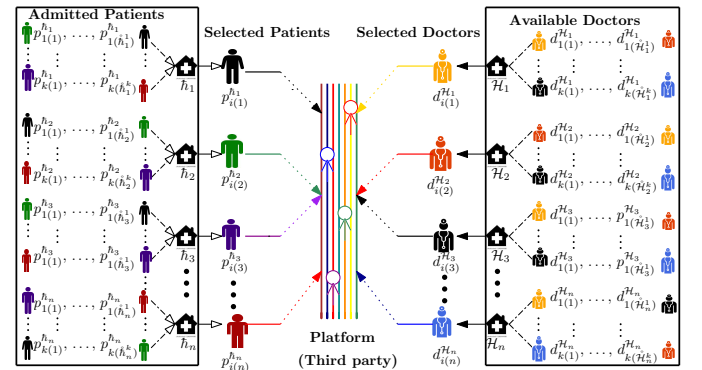


Figure 1: System model

Our model captures only a single category say  $c_i \in \mathcal{C}$  but it is to be noted that our proposed model works well for the system considering multiple categories simultaneously. Only thing is that we have to repeat the process  $k$  times as  $k$  categories are existing. Among the associated doctors to these different hospitals in a category say  $c_i$ , some of them may be interested in providing an expert consultancy to the downtrodden community for the social welfare. In

the schematic diagram shown in Figure 1, for representation purpose one doctor is selected from all the interested doctors from each hospital belonging to a particular category  $c_i \in \mathcal{C}$ . But in general one can think of the situation where multiple doctors can be selected from the available doctors from a particular hospital in a particular category  $c_i \in \mathcal{C}$  such that  $|\mathcal{P}_i| = \sum_{\mathcal{H}_j \in \mathcal{H}} \mathcal{H}_j^i$ ; where  $0 \leq \mathcal{H}_j^i \leq n$  is the number of doctors selected from hospital  $\mathcal{H}_j$  in  $c_i$  category placed into the consultancy arena. Following the above discussed criteria, a set of selected doctors in a category  $c_i$  is given as  $\mathcal{D}_i = \{d_{i(1)}^{\mathcal{H}_1}, d_{i(2)}^{\mathcal{H}_2}, \dots, d_{i(n)}^{\mathcal{H}_n}\}$  and a set of selected patients from  $c_i$  category is given as  $\mathcal{P}_i = \{p_{i(1)}^{\mathcal{H}_1}, p_{i(2)}^{\mathcal{H}_2}, \dots, p_{i(n)}^{\mathcal{H}_n}\}$ . If not specified explicitly,  $n$  denotes the total number of patients and the total number of doctors that are participating in the consultancy arena in any category  $c_i$ . Each patient  $p_{i(j)}^{\mathcal{H}_j} \in \mathcal{P}_i$  reveals a strict preference ordering over the participating set of doctors in category  $c_i \in \mathcal{C}$  i.e.  $\mathcal{D}_i$  and also each doctor  $d_{i(j)}^{\mathcal{H}_j} \in \mathcal{D}_i$  provides the strict preference ordering over the set of participating patients of category  $c_i \in \mathcal{C}$  i.e.  $\mathcal{P}_i$  in the consultancy arena. The strict preference ordering of the patient  $k$  belonging to  $c_i$  category admitted to hospital  $\mathcal{H}_j$  i.e.  $p_{i(k)}^{\mathcal{H}_j}$  over the set  $\mathcal{D}_i$  is denoted by  $\succ_k^i$ . More formally, the significance of  $d_{i(\ell)}^{\mathcal{H}_\ell} \succ_k^i d_{i(m)}^{\mathcal{H}_m}$  is that the patient  $p_{i(k)}^{\mathcal{H}_t}$  ranks doctor  $d_{i(\ell)}^{\mathcal{H}_\ell}$  above the doctor  $d_{i(m)}^{\mathcal{H}_m}$ . The preference profile of all the patients for  $k$  different categories is denoted as  $\succ = \{\succ^1, \succ^2, \dots, \succ^k\}$ , where  $\succ^i$  denotes the preference profile of all the patients in category  $c_i$  over all the doctors in set  $\mathcal{D}_i$  represented as  $\succ^i = \{\succ_1^i, \succ_2^i, \dots, \succ_n^i\}$ . The preference profile of all the patients in  $c_i$  category except the patient  $r$  is given as  $\succ_{-r}^i = \{\succ_1^i, \succ_2^i, \dots, \succ_{r-1}^i, \succ_{r+1}^i, \dots, \succ_n^i\}$ . On the other hand, the strict preference ordering of the doctor  $d_{j(t)}^{\mathcal{H}_t}$  is denoted by  $\succ_j^t$  over the set  $\mathcal{P}_j$ , where  $p_{j(\ell)}^{\mathcal{H}_\ell} \succ_j^t p_{j(m)}^{\mathcal{H}_m}$  means that doctor  $d_{j(t)}^{\mathcal{H}_t}$  ranks  $p_{j(\ell)}^{\mathcal{H}_\ell}$  above  $p_{j(m)}^{\mathcal{H}_m}$ . The set of preferences of all the doctors in  $k$  different categories is denoted as  $\succ = \{\succ_1, \succ_2, \dots, \succ_k\}$ , where  $\succ_j$  contains the strict preference ordering of all the doctors in  $c_j$  category over all the patients in set  $\mathcal{P}_j$  represented as  $\succ_j = \{\succ_j^1, \succ_j^2, \dots, \succ_j^n\}$ . The strict preference ordering of all the doctors in  $c_j$  category except the doctor  $s$  is represented as  $\succ_{-s}^j = \{\succ_j^1, \succ_j^2, \dots, \succ_j^{s-1}, \succ_j^{s+1}, \dots, \succ_j^n\}$ . Given the strict preference ordering of both the patients and the doctors, the allocation of any doctor to the patient irrespective of the category it belongs to is captured by the mapping function  $\mathcal{M} : \mathcal{P} \cup \mathcal{D} \rightarrow \mathcal{P} \cup \mathcal{D}$ . Let us denote the resulting allocation vector by  $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k\}$ ; where, each allocation vector  $\mathcal{A}_i \in \mathcal{A}$  denotes the patient-doctor pairs belonging to the  $c_i$  category denoted as  $\mathcal{A}_i = \bigcup_{j=1; k=1}^{n;n} a_{jk}^i$ , where each  $a_{lm}^i \in \mathcal{A}_i$  is a pair  $\{p_{i(l)}^{\mathcal{H}_l}, d_{i(m)}^{\mathcal{H}_m}\}$ .

#### IV. REQUIRED DEFINITIONS

This section highlights the definition of the terminologies that will be utilized throughout the paper for understanding purpose.

**Definition 1 (Blocking pair).** Fix a category  $c_k \in \mathcal{C}$ . We say that a pair  $p_{k(i)}^{\mathcal{H}_i} \in \mathcal{P}_k$  and  $d_{k(j)}^{\mathcal{H}_j} \in \mathcal{D}_k$  form a blocking pair for matching  $\mathcal{M}$ , if the following three conditions holds: (i)

$\mathcal{M}(p_{k(i)}^{\mathcal{H}_i}) \neq d_{k(j)}^{\mathcal{H}_j}$ , (ii)  $d_{k(j)}^{\mathcal{H}_j} \succ_k^i \mathcal{M}(p_{k(i)}^{\mathcal{H}_i})$ , and (iii)  $p_{k(i)}^{\mathcal{H}_i} \succ_k^j \mathcal{M}(d_{k(j)}^{\mathcal{H}_j})$ .

**Definition 2 (Stable matching).** Fix a category  $c_k \in \mathcal{C}$ . A matching  $\mathcal{M}$  is stable if there is no pair  $p_{k(i)}^{\mathcal{H}_i} \in \mathcal{P}_k$  and  $d_{k(j)}^{\mathcal{H}_j} \in \mathcal{D}_k$  such that it satisfies the conditions mentioned in (i)-(iii) in Definition 1.

**Definition 3 (Perfect matching).** Fix a category  $c_k \in \mathcal{C}$ . A matching  $\mathcal{M}$  is perfect matching if there exists one-to-one matching between the members of  $\mathcal{P}_k$  and  $\mathcal{D}_k$ .

**Definition 4 (Patient-optimal stable allocation).** Fix a category  $c_k \in \mathcal{C}$ . A matching  $\mathcal{M}$  is patient optimal if there exists no stable matching  $\mathcal{M}'$  such that  $\mathcal{M}'(p_{k(j)}^{\mathcal{H}_j}) \succ_j^k \mathcal{M}(p_{k(j)}^{\mathcal{H}_j})$  or  $\mathcal{M}'(p_{k(j)}^{\mathcal{H}_j}) = \mathcal{M}(p_{k(j)}^{\mathcal{H}_j})$  for at least one  $p_{k(j)}^{\mathcal{H}_j} \in \mathcal{P}_i$ . Similar is the situation for doctor-optimal stable matching.

**Definition 5 (Strategy-proof for requesting party).** Fix a category  $c_k$ . Given the preference profile  $\succ^k$  and  $\succ_k$  of the patients and doctors in  $c_k$  category, a mechanism  $\mathbb{M}$  is strategy-proof (truthful) for the requesting party if for each agent  $j$  of the requesting party  $a_{jk}^k \succeq \hat{a}_{jk}^k$ .

#### V. PROPOSED MECHANISMS

In this section, we provide a limelight on the *benchmark mechanism*; namely RAMHECs and the *proposed mechanism*; namely TOMHECs for the *ECs hiring problem*. The RAMHECs is given as a naive approach to our *ECs hiring problem*, that will help to better understand and analyse the more robust *dominant strategy incentive compatible (DSIC)* mechanism TOMHECs. The overview of RAMHECs is presented first along with the analysis and illustrative example and then the more robust TOMHECs is studied in detail. It is to be noted that, the further illustration of the mechanisms are done under the consideration that patient party is requesting. Moreover, one can utilize the same road map of the mechanisms by considering doctors as the requesting party. This can easily be done by just interchanging their respective roles in the mechanisms.

##### A. Randomized mechanism for hiring expert consultants (RAMHECs)

In this section, we illustrate a randomized mechanism known as RAMHECs for allocating the doctors to the patients. The idea behind proposing randomized mechanism is to better understand the more robust and philosophically strong *optimal* mechanism TOMHECs.

1) *Sketch of RAMHECs:* The idea lies behind the construction of *initialization phase* is to handle the system consisting of patients and doctors partitioned on the basis of diseases and the areas of expertise in different category set given as  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$ . The input to the *initialization phase* are the set of available patients in each category at a time i.e.  $\mathcal{P}$ , the set of available expert consultants (doctors) in each category at a time i.e.  $\mathcal{D}$ , the set of different categories in the system i.e.  $\mathcal{C}$ , the strict ordering profile of all the patients for  $k$  different categories  $\succ = \{\succ^1, \succ^2, \dots, \succ^k\}$ , and the strict

preference profile of all the doctors for  $k$  different categories  $\succsim = \{\succsim_1, \succsim_2, \dots, \succsim_k\}$ . The output of the RAMHECs is the allocation set  $\mathcal{A}$  containing the sets of all feasible allocation  $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k\}$ , for all the available categories  $i \in 1 \dots k$ . Line 2 initializes the output data structure  $\mathcal{A}$  to  $\phi$ . The *for* loop in line 3 keeps track of all the patients and doctors in each  $c_i \in \mathcal{C}$  category. Line 4 initializes all the variables and the data structures. In line 5, the *select()* returns the index of the patients set belonging to each  $c_i \in \mathcal{C}$ . In line 6, the  $\mathcal{P}^*$  data structure holds the set of patients present in  $c_i$  category at the index returned by line 5. Similarly, the *select()* in line 7, returns the index of the doctors set belonging to  $c_i$  category. The  $\mathcal{D}^*$  data structure in line 8 holds the set of doctors present at index returned by line 7. In line 9, the *while* loop keeps track of allocation of doctors to each patient. From the construction of RAMHECs, it is clear that the *while* loop terminates only when the output data structure  $\mathcal{A}_i$  in any category  $c_i \in \mathcal{C}$  contains all the patient-doctor pairs. Each of the patient is captured by the  $p^*$  data structure one by one in line 11. Line 12 checks the strict preference ordering of the returned patient from the previous line. In line 12, the doctor is randomly selected from the patient  $t$  ranking list and the index of the randomly selected doctor is held in variable  $k$ .

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**Algorithm 1** RAMHECs ( $\mathcal{D}, \mathcal{P}, \mathcal{C}, \succ, \succsim$ )

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**Output:**  $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k\}$

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1: begin
   /* Initialization phase */
2:  $\mathcal{A} \leftarrow \phi$ 
3: for each  $c_i \in \mathcal{C}$  do
4:    $k \leftarrow 0, j \leftarrow 0, d^* \leftarrow \phi, p^* \leftarrow \phi, \mathcal{A}_i \leftarrow \phi$ 
5:    $i \leftarrow \text{select}(\mathcal{P})$ 
6:    $\mathcal{P}^* \leftarrow \mathcal{P}_i$ 
7:    $i \leftarrow \text{select}(\mathcal{D})$ 
8:    $\mathcal{D}^* \leftarrow \mathcal{D}_i$ 
   /* Allocation phase */
9:   while  $|\mathcal{A}_i| \neq n$  do
10:     $t \leftarrow \text{rand}(\mathcal{P}^*)$ 
11:     $p^* \leftarrow p_{i(t)}^{h_k}$ 
12:     $k \leftarrow \text{rand}(\succsim_t^i, \mathcal{D}^*)$ 
13:     $d^* \leftarrow d_{i(k)}^{H_\ell}$ 
14:     $\mathcal{A}_i \leftarrow \mathcal{A}_i \cup \{(p^*, d^*)\}$ 
15:     $\mathcal{P}_i \leftarrow \mathcal{P}_i \setminus p^*$ 
16:     $\mathcal{D}_i \leftarrow \mathcal{D}_i \setminus d^*$ 
17:   end while
18:    $\mathcal{A} \leftarrow \mathcal{A} \cup \mathcal{A}_i$ 
19: end for
20: return  $\mathcal{A}$ 
21: end

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In line 13, the  $d^*$  data structure holds the doctor present at the index returned in line 12. Line 14 maintains the selected patient – doctor pairs in  $\mathcal{A}_i$  data structure in any  $c_i$  category. Line 15 and 16 removes the allocated patient and allocated doctors from their respective strict preference ordering. In line 18, the  $\mathcal{A}$  data structure holds the the respective set of patient-doctor pair in each category  $c_i$ . Finally, line 20 of RAMHECs

returns the final patient-doctor allocation pair set  $\mathcal{A}$  in the system.

2) *Analysis of RAMHECs:* The time taken by the *Algorithm 1* is the sum of running times for each statement executed. The time taken by line 2 is  $O(1)$ . Talking about the *for* loop, when the *for* loop executes in the usual way (i.e., due to the inner loop header), the test is executed one time more than the body of the *for* loop. In line 3, the test is executed  $(k+1)$  times, as there are  $k$  categories in our system. Line 4 – 18 will take overall  $O(n)$ ; for each execution of *for* loop. So, the overall running time will be  $O(kn)$ . Hence, the overall time complexity of *Algorithm 1* is  $O(1) + O(kn) = O(kn)$ . For smaller value of  $k$  the overall running time is linear in the size of the preference list i.e.  $O(n)$ .

3) *Correctness of RAMHECs:* The correctness of RAMHECs is proved with the loop invariant technique [32], [33]. The *loop invariant*: for each category  $c_j \in \mathcal{C}$ , at the start of each iteration of the *while* loop of lines 9 – 17, the sub-array  $\mathcal{A}_j [1..i-1]$  contains  $i-1$  patient-doctor pairs. The *loop invariant* that we have to prove is that at the end of the  $i^{\text{th}}$  iteration for a particular category  $c_j \in \mathcal{C}$  each of the  $i$  patients get one distinct doctor allocated. We must show three things for the *loop invariant* technique to be true.

**Initialization:** It is true prior to the first iteration of the *while* loop for a particular category  $c_j \in \mathcal{C}$ . Just before the first iteration of the *while* loop in  $c_j$  category  $\mathcal{A}_j \leftarrow \phi$ . This confirms that  $\mathcal{A}_j$  contains no patient-doctor pair prior to the first iteration of the *while* loop.

**Maintenance:** The *loop invariant* to be true, we have to show that if it is true before each iteration of *while* loop, it remains true before the next iteration. The body of the *while* loop allocates a doctor to a patient in a particular category i.e. each time  $\mathcal{A}_j$  is incremented by 1. Just before the  $i^{\text{th}}$  iteration, the  $\mathcal{A}_j$  data structure contains  $(i-1)$  number of patient-doctor pairs. After the  $i^{\text{th}}$  iteration, the  $\mathcal{A}_j$  data structure contains  $i$  patient-doctor pairs. This way at the end of the  $i^{\text{th}}$  iteration all the  $i$  number of patients gets a distinct doctor and the patient-doctor pairs are stored in  $\mathcal{A}_j [1..i]$ .

**Termination:** The third property is to check, what happens when the *while* loop terminates. The condition causing the *while* loop to terminate is that all the patients are allocated with one distinct doctor each in a  $c_j$  category leading to  $n$  patient doctor pair in  $\mathcal{A}_j$  data structure. Because each loop iteration increments  $\mathcal{A}_j$  by 1, we must have  $\mathcal{A}_j = n$  when all  $n$  patients are already processed. So, when the loop terminates we have a data structure  $\mathcal{A}_j [1..n]$  that has already been processed and it consists of  $n$  patient-doctor pairs.

If the RAMHECs is true for a particular category  $c_j \in \mathcal{C}$  it will remain true when all category in  $\mathcal{C}$  taken simultaneously. Hence, the RAMHECs is correct.

4) *Illustrative example:* For understanding purpose, let the category of all the patients and doctors be  $c_3$  (say eye surgery). The set of patient from 4 different hospitals  $h = \{h_1, h_2, h_3, h_4\}$  is given as:  $\mathcal{P}_3 = \{p_{3(1)}^{h_2}, p_{3(2)}^{h_3}, p_{3(3)}^{h_4}, p_{3(4)}^{h_1}\}$ . The set of available doctors engaged to 4 different hospitals  $\mathcal{H} = \{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \mathcal{H}_4\}$  is given as:  $\mathcal{D}_3 = \{d_{3(1)}^{H_3}, d_{3(2)}^{H_1}, d_{3(3)}^{H_4}, d_{3(4)}^{H_2}\}$ . The strict preference ordering revealed by the patient set  $\mathcal{P}_3$  and the doctor set  $\mathcal{D}_3$  are shown



in Figure 2a. Figure 2a can be illustrated as; say for patient  $p_{3(1)}^{\bar{h}_2}$ , the strict preference ordering of patient  $p_{3(1)}^{\bar{h}_2}$  is given as:  $d_{3(4)}^{\mathcal{H}_2} \succ_1 d_{3(3)}^{\mathcal{H}_4} \succ_1 d_{3(1)}^{\mathcal{H}_3} \succ_1 d_{3(2)}^{\mathcal{H}_1}$ . Similarly, say for doctor  $d_{3(1)}^{\mathcal{H}_3}$ , the strict preference ordering of doctor  $d_{3(1)}^{\mathcal{H}_3}$  is given as:  $p_{3(1)}^{\bar{h}_2} \succ_1 p_{3(2)}^{\bar{h}_3} \succ_1 p_{3(3)}^{\bar{h}_4} \succ_1 p_{3(4)}^{\bar{h}_1}$ .

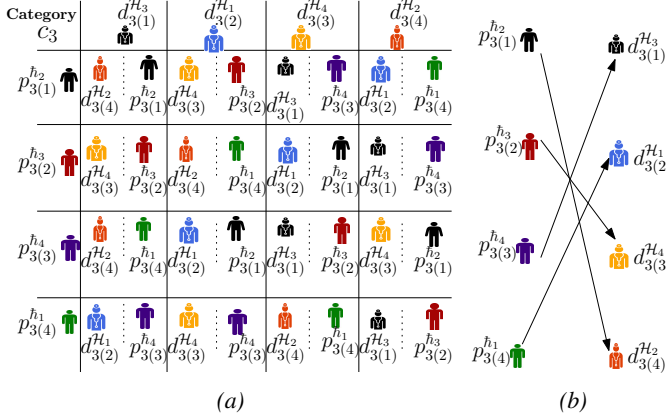


Figure 2: a. The patients and doctors provides the strict preference ordering over the available doctor set  $\mathcal{D}_3$  and the available patient set  $\mathcal{P}_3$  respectively. b. The final patient-doctor allocation pair.

Considering the set-up given in Figure. 2a, let us suppose while loop in line 9 randomly selects patient  $p_{3(3)}^{\bar{h}_4}$  from the available patients list  $\mathcal{P}_3$ . Line 12 of the RAMHECs randomly selects doctor  $d_{3(4)}^{\mathcal{H}_2}$  from the available preference ordering of  $p_{3(3)}^{\bar{h}_4}$ . At the end of first iteration of the while loop, the RAMHECs captures  $(p_{3(1)}, d_{3(4)})$  pair in the  $\mathcal{A}_i$  data structure. Similarly, in the next iteration of the while loop, the mechanism select patient; say  $p_{3(2)}^{\bar{h}_3}$  randomly from the available patients list  $\mathcal{P}_3$  and using line 12 of the RAMHECs the doctor  $d_{3(3)}^{\mathcal{H}_4}$  is randomly selected from the patient  $p_{3(2)}^{\bar{h}_3}$  preference list. In the similar fashion, the remaining allocation is done. The final patient-doctor allocation pair done by the mechanism is shown in Figure 2b.

**Observation:** It can be seen from the ranking list of  $p_{3(1)}^{\bar{h}_2}, p_{3(2)}^{\bar{h}_3}$ , and  $p_{3(4)}^{\bar{h}_1}$  that these patients do not get best doctor from the available list of doctors. The most preferred doctor by  $p_{3(1)}^{\bar{h}_2}$  is allocated to the patient  $p_{3(2)}^{\bar{h}_3}$ , whereas the most preferred doctor by  $p_{3(2)}^{\bar{h}_3}$  is allocated to  $p_{3(3)}^{\bar{h}_4}$ , and the most preferred doctor by  $p_{3(3)}^{\bar{h}_4}$  is allocated to  $p_{3(2)}^{\bar{h}_3}$ . In this scenario, the patients  $p_{3(1)}, p_{3(2)}$ , and  $p_{3(3)}$  can reallocate their current allocated doctor among themselves to make themselves better-off. Hence, from the example it can be concluded that RAMHECs is suffering from *blocking coalition*.

### B. Truthful optimal mechanism for hiring expert consultants (TOMHECs)

The idea behind proposing the TOMHECs is to overcome several non-trivial challenges such as: (a) firstly, as the patients and the doctors are *strategic* in nature, so they may misrepresent their strict preference ordering in order to gain. (b) Secondly, the allocation of the doctors done to the patient party must be *stable*. It is to noted that, the TOMHECs is *truthful*

in the sense that, the members present on the requesting side can't gain by manipulating their strict preference ordering but the party on the other side can. Along with *truthfulness*, the TOMHECs satisfies *optimality* property. The main idea of the TOMHECs is to develop a mechanism where the agents in the requesting party can't gain by manipulation.

1) *Sketch of TOMHECs:* The presence of *initialization phase* in the TOMHECs confirms the tracking of all the patients and doctors present in different categories based on the disease and area of expertise respectively. The input parameters to the TOMHECs are the set of available patients in each category at a time *i.e.*  $\mathcal{P}$ , the set of available expert consultants (doctors) in each category at a time *i.e.*  $\mathcal{D}$ , the set of different categories in the system *i.e.*  $\mathcal{C}$ , the strict ordering profile of all the patients for  $k$  different categories  $\succ = \{\succ^1, \succ^2, \dots, \succ^k\}$ , and the strict preference profile of all the doctors for  $k$  different categories  $\succ = \{\succ_1, \succ_2, \dots, \succ_k\}$ . The output of the TOMHECs is the allocation set  $\mathcal{A}$  containing the sets of all feasible allocation  $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k\}$ , for all the available categories  $i \in 1 \dots k$ . In the *initialization phase* of TOMHECs line 2 initializes the variable  $i$  to 0 and the output data structure  $\mathcal{A}$  to  $\phi$ . Considering the *allocation phase* of TOMHECs, the *for* loop in line 3 keeps track of each category  $c_i \in \mathcal{C}$ . Line 4 initializes each  $\mathcal{A}_i \in \mathcal{A}$  data structures to  $\phi$ . In line 5, the select() returns the index of the patient set belonging to particular category  $c_i \in \mathcal{C}$ . In line 6, the  $\mathcal{P}^*$  data structure holds the set of patients present in  $c_i$  category at the index returned by line 5. Similarly, the select() in line 7, returns the index of the doctors set belonging to  $c_i$  category. In line 8, the  $\mathcal{D}^*$  data structure holds the set of doctors present in  $c_i$  category at the index returned by line 7. The *for* loop in line 9-11 initializes the  $\Pi(d_{i(j)}^{\mathcal{H}_k})$  data structure to *empty* set, where  $\Pi(d_{i(j)}^{\mathcal{H}_k})$  is the data structure that keeps the track of the set  $p_{i(j)}^{\bar{h}_k} \in \mathcal{P}_i$  that propose  $d_{i(j)}^{\mathcal{H}_k}$ . The conditional check in line 12 assures that the allocation phase/operation terminates only when each of the patients is allocated a doctor in a particular category  $c_i \in \mathcal{C}$ . Line 13-16, allows each of the patients  $p_{i(j)}^{\bar{h}_k} \in \mathcal{P}_i$  to approach to the best possible doctor  $d_{i(j)}^{\mathcal{H}_k} \in \mathcal{D}_i$  that is held by  $d^*$ . Line 14 selects the most preferred doctor from the preference list of each patient  $p_{i(j)}^{\bar{h}_k} \in \mathcal{P}_i$  that is not proposed till now. In order to signify the proposal of any  $j^{th}$  patient  $p_{i(j)}^{\bar{h}_k} \in \mathcal{P}_i$  to any  $j^{th}$  doctor  $d_{i(j)}^{\mathcal{H}_k} \in \mathcal{D}_i$  a directed edge is placed from  $p_{i(j)}^{\bar{h}_k}$  to  $d_{i(j)}^{\mathcal{H}_k}$ . As stated earlier, in line 15 the  $\Pi(d_{i(j)}^{\mathcal{H}_k})$  keeps the note of the set of patient opting for any doctor  $d_{i(j)}^{\mathcal{H}_k}$ . Line 17 iterates over the engaged doctors  $d_{i(j)}^{\mathcal{H}_k}$ . In line 18, for each engaged doctor  $d_{i(j)}^{\mathcal{H}_k}$  the check is made that whether there is a multiple proposal to any doctor  $d_{i(j)}^{\mathcal{H}_k}$  or not. If the condition in line 18 is satisfied, then line 19 selects the best patient by utilizing the knowledge of preference list of doctor  $d_{i(j)}^{\mathcal{H}_k} \in \mathcal{D}_i$  and  $\Pi(d_{i(j)}^{\mathcal{H}_k})$ . The best selected patient is held in  $\mathcal{P}^*$  data structure in line 19. Line 20 checks whether the doctor  $d_{i(j)}^{\mathcal{H}_k}$  is already engaged and their in the allocation set  $\mathcal{A}_i$  with any other patient  $p_{i(j)}^{\bar{h}_k} \in \mathcal{P}_i$ . If this is the case, then the check is made that, if the patient in  $\mathcal{P}^*$  data structure is preferred over the already engaged patient  $p_{i(j)}^{\bar{h}_k}$ . The already available  $(p_{i(j)}^{\bar{h}_k}, d_{i(j)}^{\mathcal{H}_k})$  pair is removed from the  $\mathcal{A}_i$  as depicted

in line 21. In line 22, the new patient-doctor pair is maintained in  $\mathcal{A}_i$  data structure. In line 23, all the patients are removed except the currently allocated patient in line 22.

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**Algorithm 2** TOMHECs ( $\mathcal{D}, \mathcal{P}, \mathcal{C}, \succ, \asymp$ )

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**Output:**  $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k\}$

```

1: begin
  /* Initialization phase */
2:  $i \leftarrow 0, \mathcal{A} \leftarrow \phi$ 
  /* Allocation phase */
3: for each  $c_i \in \mathcal{C}$  do
4:    $\mathcal{A}_i \leftarrow \phi$ 
5:    $i \leftarrow \text{select}(\mathcal{P})$ 
6:    $\mathcal{P}^* \leftarrow \mathcal{P}_i$ 
7:    $i \leftarrow \text{select}(\mathcal{D})$ 
8:    $\mathcal{D}^* \leftarrow \mathcal{D}_i$ 
9:   for each  $d_{i(j)}^{\mathcal{H}_k} \in \mathcal{D}^*$  do
10:     $\Pi(d_{i(j)}^{\mathcal{H}_k}) \leftarrow \phi$   $\triangleright \Pi(d_{i(j)}^{\mathcal{H}_k})$  data structure keeps
    track of set of  $p_{i(j)}^{\mathcal{H}_k} \in \mathcal{P}_i$  requesting to  $d_{i(j)}^{\mathcal{H}_k}$ .
11:   end for
12:   while  $|\mathcal{A}_i| \neq n$  do
13:     for each free patient  $p_{i(j)}^{\mathcal{H}_k} \in \mathcal{P}_i$  do
14:        $d^* \leftarrow$  select the most preferred doctor from  $\succ_i^j$ 
       not approached till now.
15:        $\Pi(d^*) \leftarrow \Pi(d^*) \cup p_{i(j)}^{\mathcal{H}_k}$ 
16:     end for
17:     for each engaged doctor  $d_{i(j)}^{\mathcal{H}_k} \in \mathcal{D}_i$  do
18:       if  $|\Pi(d_{i(j)}^{\mathcal{H}_k})| > 1$  then
19:          $p^* \leftarrow \text{select\_best}(\succ_i^j, \Pi(d_{i(j)}^{\mathcal{H}_k}))$ 
20:         if  $(p_{i(j)}^{\mathcal{H}_k}, d_{i(j)}^{\mathcal{H}_k}) \in \mathcal{A}_i$  and  $p^* \succ_i^j p_{i(j)}^{\mathcal{H}_k}$  then
21:            $\mathcal{A}_i \leftarrow \mathcal{A}_i \setminus (p_{i(j)}^{\mathcal{H}_k}, d_{i(j)}^{\mathcal{H}_k})$ 
22:            $\mathcal{A}_i \leftarrow \mathcal{A}_i \cup (p^*, d_{i(j)}^{\mathcal{H}_k})$ 
23:            $\Pi(d_{i(j)}^{\mathcal{H}_k}) \leftarrow \Pi(d_{i(j)}^{\mathcal{H}_k}) \setminus \Pi(d_{i(j)}^{\mathcal{H}_k}) - \{p^*\}$ 
24:         else if  $(p_{i(j)}^{\mathcal{H}_k}, d_{i(j)}^{\mathcal{H}_k}) \notin \mathcal{A}_i$  then
25:            $\mathcal{A}_i \leftarrow \mathcal{A}_i \cup (p^*, d_{i(j)}^{\mathcal{H}_k})$ 
26:            $\Pi(d_{i(j)}^{\mathcal{H}_k}) \leftarrow \Pi(d_{i(j)}^{\mathcal{H}_k}) \setminus \Pi(d_{i(j)}^{\mathcal{H}_k}) - \{p^*\}$ 
27:         end if
28:       else if  $|\Pi(d_{i(j)}^{\mathcal{H}_k})| == 1$  then
29:         if  $(\Pi(d_{i(j)}^{\mathcal{H}_k}), d_{i(j)}^{\mathcal{H}_k}) \notin \mathcal{A}_i$  then
30:            $\mathcal{A}_i \leftarrow \mathcal{A}_i \cup (\Pi(d_{i(j)}^{\mathcal{H}_k}), d_{i(j)}^{\mathcal{H}_k})$ 
31:         end if
32:       end if
33:     end for
34:   end while
35:    $\mathcal{A} \leftarrow \mathcal{A} \cup \mathcal{A}_i$ 
36: end for
37: return  $\mathcal{A}$ 
38: end

```

---

Now, it may be the case that the doctor  $d_{i(j)}^{\mathcal{H}_k}$  is occurring first time in the allocation list with the most preferred patient say  $p^*$  that check is guaranteed by the line 24. If the check in the line 24 is satisfied then, the doctor  $d_{i(j)}^{\mathcal{H}_k}$  is added in the allocation vector along with the most preferred patient  $p^*$  using line 25. Line 26 removes all the patients requesting to  $d_{i(j)}^{\mathcal{H}_k}$

except the most preferred patient  $p^*$ . Line 28-32 keeps track of all the doctors  $d_{i(j)}^{\mathcal{H}_k}$  to which only one patient is approaching. In line 30, the  $\mathcal{A}_i$  data structure holds the respective set of patient-doctor pairs in each category  $c_i$ . Line 35, keeps track of allocation vectors of each category  $c_i \in \mathcal{C}$  in  $\mathcal{A}$ . Finally, line 37 of TOMHECs returns the final patient-doctor allocation pairs in the system.

**Proposition 1.** For the system consisting of  $k$  different categories, the number of iterations required to determine the stable allocation using TOMHECs is bounded above by  $kn^2$ .

*Proof.* For each category  $c_i \in \mathcal{C}$ , it is easy to prove that, the number of iterations required for determining the stable allocation using TOMHECs is bounded above by  $n^2$ . The TOMHECs is constructed in such a way that each of the member in the requesting party (say *patient party*) will access his/her (henceforth his) preference list in descending order of interest. Talking about the particular category  $c_i$ , the preference list of each patient contains  $n$  distinct doctors and due to the nature of TOMHECs, each of the members of *patient party* can show his willingness towards any doctor not more than once. So, it can be concluded that, in worst case, each of the patient can show his willingness to  $n$  distinct doctors. As there are  $n$  distinct patients in the patient party and each may move down to requesting all the  $n$  distinct doctors in their preference list. This will lead to a total of  $n^2$  proposals in a particular category  $c_i$  in worst case.

Hence, it is proved that the TOMHECs is bounded above by  $n^2$  number of iterations for determining the stable allocation in a particular category  $c_i$ . As we have  $k$  category  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$ , from our claim it must be true for any category  $c_j \in \mathcal{C}$ . So, in worst case our system may perform a total of  $kn^2$  iterations for determining the stable allocation. ■

2) *Correctness of the TOMHECs:* The correctness of the TOMHECs is proved with the *loop invariant* technique [32] [33].

The *loop invariant*: Fix a category  $c_i$ . At the start of  $\ell^{th}$  iteration of the *while* loop, the number of temporarily processed patient-doctor pairs or in other words the number of patient-doctor pair held by  $\mathcal{A}_i$  in any  $c_i$  category is given as:  $|\cup_{j=1}^{\ell-1} \mathcal{A}'_j|$ , where  $\mathcal{A}'_j$  is the net patient-doctor pairs temporarily added in the set  $\mathcal{A}'_j$  at the  $j^{th}$  iteration. So, on average the number of patients or doctors (whomsoever is greater) that are to be explored are  $n - |\cup_{j=1}^{\ell-1} \mathcal{A}'_j|$ . From the construction of the TOMHECs it is clear that after any  $\ell^{th}$  iteration this condition holds:  $0 \leq n - |\cup_{j=1}^{\ell} \mathcal{A}'_j| \leq n$ ; where  $1 \leq \ell \leq n^2$ . The net minimum number of patient-doctor pairs that are processed temporarily at any  $k^{th}$  iteration is *zero*. Hence, inequality  $0 \leq n - |\cup_{j=1}^{\ell} \mathcal{A}'_j| \leq n$  is always *true*. We must show three things for this *loop invariant* to be true.

**Initialization:** It is true prior to the first iteration of the *while* loop. Just before the first iteration of the *while* loop, in TOMHECs the inequality  $0 \leq n - |\cup_{j=1}^{\ell} \mathcal{A}'_j| \leq n$  blows down to  $0 \leq n - 0 \leq n \Rightarrow 0 \leq n \leq n$  i.e. no patient-doctor pair is temporarily added to  $\mathcal{A}_i$  apriori i.e. before the first iteration of

while loop. This confirms that  $\mathcal{A}_i$  contains no patient-doctor pair.

**Maintenance:** For the *loop invariant* to be true, if it is true before each iteration of the *while* loop, it remains true before the next iteration of the *while* loop. The body of *while* loop allocates doctor(s) to the patient(s) with each doctor is allocated to a patient; *i.e.* each time the cardinality of  $\mathcal{A}_i$  is either incremented by some amount or remains similar to previous iteration. Just before the  $\ell^{th}$  iteration the patient-doctor pairs temporarily added to  $\mathcal{A}_i$  are  $\bigcup_{j=1}^{\ell-1} \mathcal{A}'_j$ . So, one can conclude from here that the number of patient-doctor pairs that are left is given by inequality:  $0 \leq n - |\bigcup_{j=1}^{\ell-1} \mathcal{A}'_j| \leq n$ . After the  $(\ell - 1)^{th}$  iteration, the available number of patient-doctor pair  $n - |\bigcup_{j=1}^{\ell-1} \mathcal{A}'_j| \geq 0$  can be captured under two cases:

**Case 1:** If  $|\mathcal{A}_i| = n$ :

This case will lead to exhaust all the remaining patient-doctor pair in the current  $\ell^{th}$  iteration and no patient-doctor pair is left for the next iteration. The inequality  $n - (|\bigcup_{j=1}^{\ell-1} \mathcal{A}'_j| \cup |\mathcal{A}'_\ell|) = n - (|\bigcup_{j=1}^{\ell} \mathcal{A}'_j|) = n - |\mathcal{A}_i| = 0$ . Hence, it means that all the remaining patient-doctor is absorbed in this iteration and no patient-doctor pair is left for processing.

**Case 2:** If  $|\mathcal{A}_i| < n$ :

This case captures the possibility that there may be the scenario when few patient-doctor pairs from the remaining patient-doctor pairs may still left out; leaving behind some of the pairs for further iterations. So, the inequality  $n - (|\bigcup_{j=1}^{\ell-1} \mathcal{A}'_j| \cup |\mathcal{A}'_\ell|) > 0 \Rightarrow n > n - (|\bigcup_{j=1}^{\ell-1} \mathcal{A}'_j|) > 0$  is satisfied.

From Case 1 and Case 2, at the end of  $\ell^{th}$  iteration the loop invariant is satisfied.

**Termination:** It is clear that in each iteration the cardinality of output data structure *i.e.*  $\mathcal{A}_i$  either incremented by some amount or remains as the previous iteration. This indicates that at some  $\ell^{th}$  iteration the loop terminates by dissatisfying the condition of the while loop  $|\mathcal{A}_i| \neq n$  at line 12. When the loop terminates it is for sure that  $|\mathcal{A}_i| = n$ . We can say  $n - |\bigcup_{j=1}^{\ell} \mathcal{A}'_j| = 0 \Rightarrow 0 \leq n$ . Thus, this inequality indicates that all the  $n$  patient and doctors in  $c_i$  category are processed and each patient allocated a best possible doctor when the loop terminates.

If the TOMHECs is true for the  $c_i \in \mathcal{C}$  category it will remain true when all category in  $\mathcal{C}$  taken simultaneously. Hence, the TOMHECs is correct.

3) *Illustrative example:* For example purpose, considering the set-up shown in Figure 3a similar to Figure 2a. So, according to line 13-16 of TOMHECs each of the patients  $p_{3(1)}^{h_2}$ ,  $p_{3(2)}^{h_3}$ ,  $p_{3(3)}^{h_4}$ , and  $p_{3(4)}^{h_1}$  are requesting to the most preferred doctor from their respective preference list. In the next step, we will check if any requested doctor among  $d_{3(1)}^{h_2}$ ,  $d_{3(2)}^{h_3}$ ,  $d_{3(3)}^{h_4}$ , and  $d_{3(4)}^{h_1}$  has got the multiple request from the patients in  $\mathcal{P}_3$  shown in Figure 3b. Now, it can be seen that, in the first iteration of TOMHECs doctor  $d_{3(4)}^{h_1}$  have got requests from patients  $p_{3(1)}^{h_2}$ , and  $p_{3(3)}^{h_4}$ . As each doctor can be assigned to only one patient, so this competitive environment between patient  $p_{3(1)}^{h_2}$ , and  $p_{3(3)}^{h_4}$  can be resolved by considering the strict preference ordering of doctor  $d_{3(4)}^{h_1}$  over the available patients in  $\mathcal{P}_3$ . From the strict preference ordering of doctor  $d_{3(4)}^{h_1}$  it is clear that patient  $p_{3(3)}^{h_4}$  is preferred over patient  $p_{3(1)}^{h_2}$ .

Hence, patient  $p_{3(1)}^{h_2}$  is rejected. So, for the meanwhile  $p_{3(2)}^{h_3}$  gets a doctor  $d_{3(3)}^{h_4}$ ,  $p_{3(3)}^{h_4}$  gets a doctor  $d_{3(4)}^{h_1}$ , and  $p_{3(4)}^{h_1}$  gets a doctor  $d_{3(2)}^{h_3}$ . Now, as the patient  $p_{3(1)}^{h_2}$  do not get his/her (henceforth his) most preferred doctor *i.e.*  $d_{3(4)}^{h_1}$  from his preference list. So, he will request the second best doctor *i.e.*  $d_{3(3)}^{h_4}$  from his preference list. As doctor  $d_{3(3)}^{h_4}$  is already been requested by  $p_{3(2)}^{h_3}$ , the similar situation now occurs in case of doctor  $d_{3(3)}^{h_4}$  where patients  $p_{3(1)}^{h_2}$  and  $p_{3(2)}^{h_3}$  are simultaneously requesting to doctor  $d_{3(3)}^{h_4}$  as shown in Figure 3c. Looking at the preference list of  $d_{3(3)}^{h_4}$ , we get, patient  $p_{3(1)}^{h_2}$  is preferred over  $p_{3(2)}^{h_3}$ . So, patient  $p_{3(2)}^{h_3}$  is rejected. Now,  $p_{3(2)}^{h_3}$  request the second best doctor *i.e.*  $d_{3(4)}^{h_1}$  from his preference list. Here, it can be seen that all the patient-doctor pairs are matched. This allocation is a *stable* allocation. Here, the patient  $p_{3(4)}^{h_1}$  and  $p_{3(3)}^{h_4}$  ends up with their first choice whereas the patient  $p_{3(1)}^{h_2}$  and  $p_{3(2)}^{h_3}$  ends up with their second choice as shown in Figure 3d.

### C. Several properties

The proposed TOMHECs has several compelling properties. These properties are discussed next.

**Proposition 2.** *The matching computed by the Gale-Shapley mechanism [20] [21] results in a stable matching.*

**Proposition 3.** *A stable matching computed by Gale-Shapley mechanism [20] [21] is requesting party optimal.*

**Proposition 4.** *Gale-Shapley mechanism [20] [21] is truthful for the requesting party.*

Following the above mentioned propositions and motivated by [20] [21] we have proved that the TOMHECs results in *stable*, *optimal*, and *truthful* allocation when all the  $k$  different categories are taken simultaneously.

**Lemma 1 (Requesting party stability).** *TOMHECs results in a stable allocation for the requesting party (patient party or doctor party).*

*Proof.* Fix a category  $c_i \in \mathcal{C}$ . Let us suppose for the sake of contradiction there exists a *blocking pair*  $(p_{i(j)}^{h_k}, d_{i(j)}^{h_l})$  that results in an unstable matching  $\mathcal{M}$  for the requesting party. As there exists a *blocking pair*  $(p_{i(j)}^{h_k}, d_{i(j)}^{h_l})$  it may be due to the case that  $(p_{i(j)}^{h_k}, d_{i(k)}^{h_j})$  and  $(p_{i(k)}^{h_j}, d_{i(j)}^{h_l})$  are their in the resultant matching  $\mathcal{M}$ . This situation will arise only when  $d_{i(j)}^{h_l} \succ_j^i d_{i(k)}^{h_j}$  *i.e.* in the strict preference ordering of patient  $p_{i(j)}^{h_k}$  doctor  $d_{i(j)}^{h_l}$  is preferred over doctor  $d_{i(k)}^{h_j}$ . From the matching result  $\mathcal{M}$  obtained, it can be seen that in spite of the fact that  $d_{i(j)}^{h_l} \succ_j^i d_{i(k)}^{h_j}$ ;  $d_{i(j)}^{h_l}$  is not matched with  $p_{i(j)}^{h_k}$  by the TOMHECs. So, this upset may happen only when doctor  $d_{i(j)}^{h_l}$  received a proposal from a patient  $p_{i(k)}^{h_j}$  to whom  $d_{i(j)}^{h_l}$  prefers over  $p_{i(j)}^{h_k}$  *i.e.*  $p_{i(k)}^{h_j} \succ_i^j p_{i(j)}^{h_k}$ . Hence, this contradicts the fact that the  $(p_{i(j)}^{h_k}, d_{i(j)}^{h_l})$  is a *blocking pair*. As there exists no *blocking pair*, it can be said that the resultant matching by TOMHECs is *stable*.

From our claim that, the TOMHECs results in a *stable*



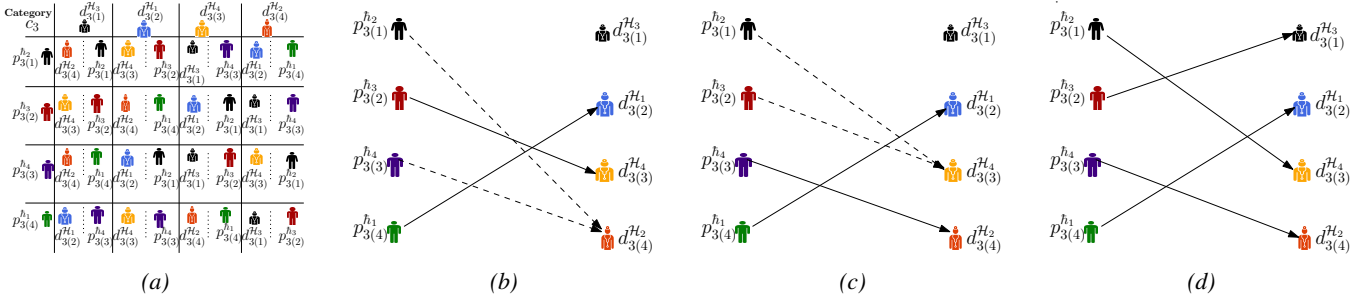


Figure 3: The progress of the proposed mechanism on the set of  $n = 4$  patients and  $n = 4$  doctors belonging to  $c_3$  category. a. The patients and the doctors belonging to  $c_3$  category provides the strict preference over the available doctors set  $\mathcal{D}_3$  and available patients set  $\mathcal{P}_3$  respectively. b. Based on the preference of the patients each patient first preference is taken into consideration for constructing the partial graph. c. Now the doctor getting the multiple request will check the preference list of his/him. d. The final allocation of patient-doctor pair.

matching in a particular category  $c_i$ , it must be true for any category. Hence, it must be true for the system considering the  $k$  categories simultaneously. ■

**Lemma 2 (Requesting party optimality).** *A stable allocation resulted by TOMHECs is requesting party (patient or doctor) optimal.*

*Proof.* Fix a category  $c_i \in \mathcal{C}$ . Let us suppose for the sake of contradiction that the allocation set  $\mathcal{A}_i$  obtained using TOMHECs is not an *optimal* allocation for requesting party (say patient party). Then, from **Lemma 1** there exists a *stable* allocation  $\mathcal{M}'$  such that  $\mathcal{M}'(p_{i(j)}) \succ_j^i \mathcal{M}(p_{i(j)})$  or  $\mathcal{M}'(p_{i(j)}) = \mathcal{M}(p_{i(j)})$  for all patients in  $\mathcal{P}_i$  with  $\mathcal{M}'(p_{i(j)}) \succ_j^i \mathcal{M}(p_{i(j)})$  for at least one patient in  $\mathcal{P}_i$ . Therefore, it must be the case that where some patient  $p_{i(j)}^{h_k}$  proposes to  $\mathcal{M}(p_{i(j)})$  before  $\mathcal{M}'(p_{i(j)})$  since  $\mathcal{M}'(p_{i(j)}) \succ_j^i \mathcal{M}(p_{i(j)})$  and is rejected by  $\mathcal{M}'(p_{i(j)})$ . Since doctor  $\mathcal{M}'(p_{i(j)})$  rejects patient  $p_{i(j)}^{h_k}$ , the doctor  $\mathcal{M}'(p_{i(j)})$  must have received a better proposal from a patient  $p_{i(k)}^{h_j}$  to whom doctor  $\mathcal{M}'(p_{i(j)})$  prefers over  $p_{i(j)}^{h_k}$  i.e.  $p_{i(k)}^{h_j} \succ_i^j p_{i(j)}^{h_k}$ . Since this is the first iteration at which a doctor rejects a patient under  $\mathcal{M}'$ . It follows that  $\mathcal{M}'(p_{i(j)}) \succ_j^i \mathcal{M}(p_{i(j)})$ . ■

## VI. EXPERIMENTAL FINDINGS

The experiments are carried out in this section to compare the efficacy of the TOMHECs based on the data (i.e. preference lists of the doctors and patients) generated randomly. The *randomized* mechanism also known as a *naive mechanism* i.e. RAMHECs is considered as the benchmark mechanism. It is to be noted that RAMHECs do not results in *optimal* allocation. In our simulation results, the lack of *optimality* in allocation by RAMHECs can be seen evidently. The experiments are done using Python. Numpy library of Python is used to generate the data randomly.

### A. Simulation setup

In this paper, we have proposed a *truthful* and *optimal* mechanism for allocating a doctor to a patient in several different categories under non-monetary environment. For the simulation purpose, the number of available patients and

doctors say  $n$  is bounded above by  $n = 600$ . In simulation,  $n$  value is taken as:  $n = 100, n = 200, n = 300, n = 400, n = 500$ , and  $n = 600$ . The strict preference ordering of each patient and doctor are generated randomly. It is to be noted that the simulation is done for a particular category say  $c_i \in \mathcal{C}$ .

### B. Performance metrics

The efficacy of TOMHECs is measured under the banner of two important parameters:

a) **Satisfaction level ( $\eta_\ell$ ):** It is defined as the sum over the difference between the index of the doctor (patient) allocated from the patient's (doctor's) preference list to the index of the most preferred doctor (patient) by the patient (doctor) from his/her preference list. Considering the *requesting party*, the  $\eta_\ell^j$  for  $c_j$  category is defined as:

$$\eta_\ell^j = \sum_{i=1}^n \left( \bar{\xi}_i - \xi_i \right) \quad (1)$$

Where,  $\bar{\xi}_i$  is the index of the doctor (patient) allocated from the initially provided preference list of the patients (doctors)  $i$ , and  $\xi_i$  is the index of the most preferred doctor (patient) in the initially provided preference list of patient (doctor)  $i$ . Equation 1 can also be utilized to calculate the satisfaction level of the *requested party* in  $c_j$  category. If the system consists of  $k$  different categories i.e.  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$ , then the total satisfaction level ( $\eta_\ell$ ) of the system is given as:

$$\eta_\ell = \sum_{j=1}^k \eta_\ell^j \quad (2)$$

$$\eta_\ell = \sum_{j=1}^k \sum_{i=1}^n \left( \bar{\xi}_i - \xi_i \right) \quad (3)$$

Here,  $\eta_\ell$  of the system means that the total satisfaction level incurred either in *requesting party* or in *requested party* for all the available category  $c_i \in \mathcal{C}$ . It is to be noted that lesser the value of satisfaction level higher will be the satisfaction of patients or doctors.

**Definition 6.** *In any category  $c_i$ , the satisfaction level ( $\eta_\ell^i$ ) is bounded above by the number of iterations in TOMHECs i.e.  $n^2$ . Hence, the total satisfaction level ( $\eta_\ell$ ) of the system*

consisting of  $k$  different categories is be bounded above by  $kn^2$ .

**b) Number of preferable allocation ( $\zeta_i$ ):** The term "preferable allocation" refers to the allocation of most preferred doctor or patient from the revealed preference lists by the patients or the doctors respectively. For a particular patient or doctor the *preferable allocation* is captured by the function  $f : \mathcal{P}_i \rightarrow \{0, 1\}$ . For the category  $c_i$ , the number of preferable allocation is defined as the number of patients (doctors) getting their first choice from the initially provided preference list. Mathematically,

$$\zeta_i = \sum_{j=1}^n f(p_{i(j)}^{\bar{h}_k}) \quad (4)$$

Considering the overall available categories  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$ , the total number of preferable allocation of the system is given as:

$$\zeta = \sum_{i=1}^k \zeta_i = \sum_{i=1}^k \sum_{j=1}^n f(p_{i(j)}^{\bar{h}_k}) \quad (5)$$

### C. Simulation directions

The three direction are seen for measuring the performance of TOMHECs, they are:

- Fix a category  $c_i \in \mathcal{C}$ , when all the patients and doctors are reporting their true preference list.
- When all the members of the *requesting party* (say patients) are revealing their true preference ordering but the members of the *requested party* (say doctors) are misreporting their preference lists.
- When all the members of the *requested party* (say doctors) are revealing their true preference ordering but the members of the *requesting party* (say patients) are misreporting their preference lists.

### D. Result analysis

Our simulation results are analysed by considering the agenda of *truthful* and *non-truthful* revelation of the preference ordering by the patients and the doctors. First, the two parameters for a particular category  $c_i \in \mathcal{C}$ :  $\eta_\ell^i$  and  $\zeta_i$  are discussed under the banner of "*truthful revelation of the preference ordering*" i.e. no member of the requesting party or requested party will vary his preference list. Next, these stated parameters ( $\eta_\ell^i$  and  $\zeta_i$ ) are studied under the manipulative environment of the market. In this section, the result is simulated for the above mentioned three cases and discussed.

**Expected number of patients/doctors deviating:** As the doctors and patients present in our system are assumed to be *strategic* in nature. So, it may be the case that the patients/doctors may misreport their true preference ordering in order to be benefited. In this section, we wish to compute the expected number of patients/doctors among total available patients/doctors in the system in a particular category  $c_i$  deviating from their true preference ordering. The analysis is motivated by [32]. The following analysis using indicator

random variable justifies the idea of choosing parameters of deviation (i.e. 1/2, 1/4, 1/8).

Let  $\chi_j$  be the random variable associated with the event  $e_{ji}$ ; in which  $j^{th}$  patient in  $c_i$  category varies its preference ordering. This variable counts the number of patients/doctors misreporting their true preference ordering, and it is 1 if a patient/doctor misreports and 0 otherwise. Then, the *indicator random variable* associated with  $e_{ji}$  is defined as:

$$I\{e_{ji}\} = \begin{cases} 1 & \text{if } e_{ji} \text{ occurs} \\ 0 & \text{Otherwise} \end{cases}$$

Thus, we can write

$$\chi_j = I\{e_{ji}\}$$

$$\chi_j = \begin{cases} 1 & \text{if } e_{ji} \text{ occurs} \\ 0 & \text{Otherwise} \end{cases}$$

The expected number of patients/doctors misreporting their true preference ordering is simply the expected value of our indicator variable  $\chi_j$ :

$$E[\chi_j] = E[I\{e_{ji}\}]$$

From the definition of expectation we can write;

$$E[\chi_j] = 1 \cdot Pr\{e_{ji}\} + 0 \cdot Pr\{\bar{e}_{ji}\} = 1 \cdot Pr\{e_{ji}\} = 1/2$$

It is to be noted that given any patient/doctor whether he will misreport his true preference list is taken as 1/2. Let  $\chi$  be the random variable denoting the total number of patients/doctors varying their preference ordering in a particular category  $c_i \in \mathcal{C}$ . By utilizing the properties of random variable, it can be written as

$$\chi = \sum_{j=1}^n \chi_j$$

Taking expectation both side:

$$E[\chi] = E\left[\sum_{j=1}^n \chi_j\right]$$

By the property of linearity of expectation, we can write;

$$E[\chi] = \sum_{j=1}^n E[\chi_j] = \sum_{j=1}^n 1/2 = n/2$$

Similarly, given any patient/doctor whether he will misreport his true preference ordering is taken as 1/4 and 1/8, then the expected number of patients/doctors that may vary their true preference ordering becomes  $n/4$  and  $n/8$  respectively.

Our result analysis is broadly classified into two categories:

• **Case 1: Patient party requesting** Under the consideration that the patients and doctors are reporting their true preference ordering, it can be seen in Figure. 4 and Figure. 5 that the satisfaction level and the number of best allocation respectively of the patients using TOMHECs (TOMHECs-P) is more than the satisfaction level and number of best allocation of the patients using RAMHECs (RAMHECs-P). When the manipulative nature of the members of the patient party is considered then it is found that the  $\eta_\ell^j$  (for  $c_j$  category) and number of best allocation in case of TOMHECs

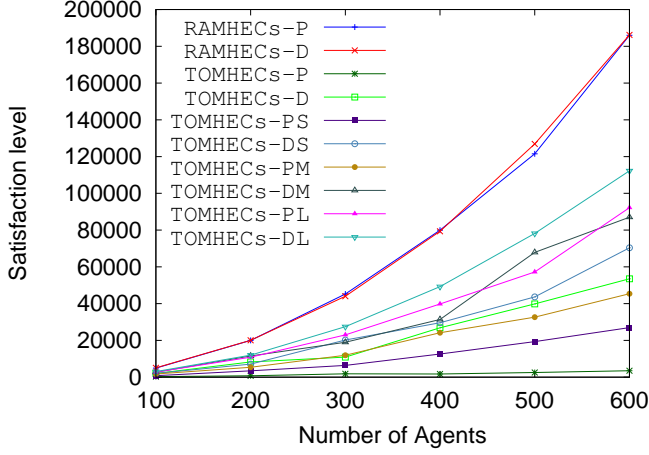


Figure 4: Comparison of satisfaction level with patients requesting

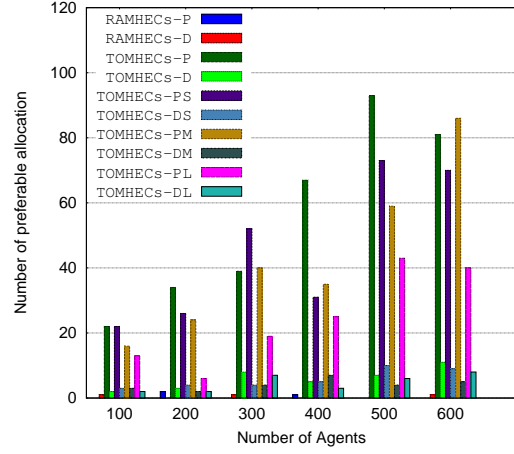


Figure 5: Comparison of number of preferable allocation with patients requesting

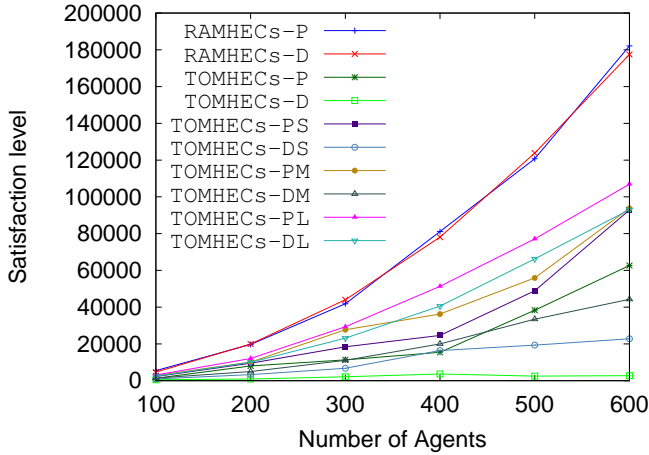


Figure 6: Comparison of satisfaction level with doctors requesting

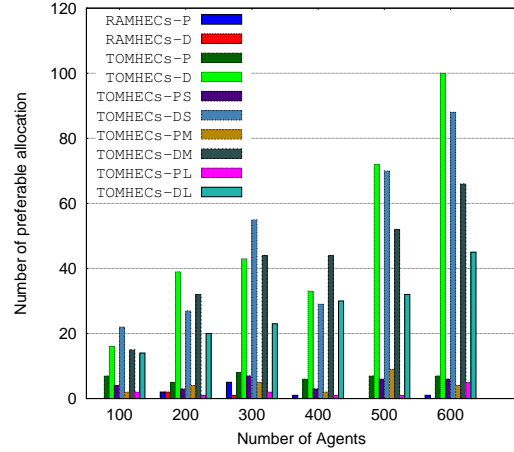


Figure 7: Comparison of number of preferable allocation with doctors requesting

is more than the  $\eta_{\ell}^j$  of the patients in case of TOMHECs with small variation (TOMHECs-PS) is more than the  $\eta_{\ell}^j$  and number of best allocation of the patients in case of TOMHECs with medium variation (TOMHECs-PM) and is more than the  $\eta_{\ell}^j$  and number of best allocation of the patients in case of TOMHECs with large variation (TOMHECs-PL). As it is intuitive from the construction of TOMHECs because TOMHECs is not vulnerable to manipulation if the patients are varying their preference ordering, then the patients will not be allocated doctors better than the doctors allocated to the patients when reporting their true preference ordering.

• **Case 2: Doctor party requesting** In this it can be seen in Figure. 6 and Figure. 7 that the satisfaction level and the number of best allocation respectively of the doctors using TOMHECs (TOMHECs-D) is more than the satisfaction level and number of best allocation of the doctors using RAMHECs (RAMHECs-D). When the manipulative nature of the members of the doctor party is considered then it is found that the  $\eta_{\ell}^j$  (for  $c_j$  category) and number of best allocation in case of TOMHECs is more than the  $\eta_{\ell}^j$  and

number of best allocation of the doctors in case of TOMHECs with small variation (TOMHECs-DS) is more than the  $\eta_{\ell}^j$  and number of best allocation of the doctors in case of TOMHECs with medium variation (TOMHECs-DM) and is more than the  $\eta_{\ell}^j$  and number of best allocation of the doctors in case of TOMHECs with large variation (TOMHECs-DL).

From the above two discussed cases, one can easily see the *optimality* property of the TOMHECs for the *requesting party*.

## VII. CONCLUSIONS AND FUTURE WORKS

We have tried to model the *ECs hiring problem* as a two sided matching problem. It turns out that the *ECs hiring problem* studied in this paper present the interesting connection to two of the best studied problems in two sided matching market, namely *stable marriage problem* and *job recruitment system*. We believe that as our work has practical appeal as well as while it is interesting, on its own, from the healthcare point of view. This paper proposed an *optimal* and *truthful* mechanism, namely TOMHECs to allocate the ECs to the patients.

A potential and natural direction for future work is to study *ECs hiring problem* under various other settings. Several obvious more general settings are of  $n$  patients and  $m$  doctors ( $m \neq n$  or  $m = n$ ) with some of the available patients and doctors, instead of providing the strict preference ordering over all the available doctors and patients respectively, may reveal their strict preference ordering over the subset of available doctors and patients respectively. The other interesting direction is to study the discussed set-ups with the presence of *ties* in the preference ordering of the patients and doctors.

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